On reversing selected performance indicators used to evaluate a set of business units

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Introduction

Preliminary reversing of some of the indicators is a very useful approach

- when ordering business units, described by a set of many performance indicators
- reversing provides a new set of traits, with as much of them, as possible strongly positively correlated
- thus the sum of new traits will have greater variability than before reversing

Aims and conjectures

- to detect the latent trait which governs the ranking, and to measure variability of that trait
- the stronger is this variability, the more meaningful is the ranking
- variability should be null when all business units perform „equally well”, and should increase along with departure from equality

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Similarity measure

- for **one** indicator of performance measured on the ratio scale, its variability can be evaluated by the *Gini index*
- for **more than one** indicator measured on the ratio scale it is necessary to transform the indicators to be “possibly most similar” and then use the *Gini index* of the sum of transformed indicators as their *total similarity measure* (or as the *variability* of the sum)

Bank Example - description

Performance indicators for 5 banks:

- X_1 = *ROE* (Return of Equity)
- X_2 = *ROA* (Return of Assets)
- X_3 = *P/T* (Personal Costs in Total Costs)
- X_4 = *C/I* (Costs per Income)
- *Z = sum of ranks for each bank*

For this table *Gini index* for the sum of ranks equals:

- $Gini(Z(T_1)) = 0.0867$

Table T_1

	X1	X2	X3	X4	Z
B1	5	4	1	1	11
B2	4	5	3	2	14
B3	3	3	5	3	14
B4	2	2	4	4	12
B5	1	1	2	5	9

Dissimilarity measure

There is well known dissimilarity measure for set of variables, called *maximal Spearman rho* and denoted ρ^*_{max}

(see: T.Kowalczyk, E.Pleszczyńska, F.Ruland (eds.) "*Grade Models and Methods for Data Analysis*", Section 8.6)

For this table (and this ordering of rows and columns)

maximal Spearman Rho equals:

$$\rho^*_{max}(T_1) = 0.3829$$

Table T_1

	X1	X2	X3	X4	Z
B1	5	4	1	1	11
B2	4	5	3	2	14
B3	3	3	5	3	14
B4	2	2	4	4	12
B5	1	1	2	5	9

Bank Example – before reversing

Table T_1

	X1	X2	X3	X4	Z
B1	5	4	1	1	11
B2	4	5	3	2	14
B3	3	3	5	3	14
B4	2	2	4	4	12
B5	1	1	2	5	9

table T_1 with initial rows ordering

- $Gini(Z(T_1)) = 0.0867$
- $\rho^*_{max}(T_1) = 0.3829$

Table T_1

	X1	X2	X3	X4	Z
B3	3	3	5	3	14
B2	4	5	3	2	14
B4	2	2	4	4	12
B1	5	4	1	1	11
B5	1	1	2	5	9

table T_1 with rows ordered according to the ranks sum Z

- $Gini(Z(T_1)) = 0.0867$
- $\rho^*_{max}(T_1) = 0.0654$

Bank Example – 1st reversing

Table T_2

	X1	X2	X3	rev(X4)	Z
B1	5	4	1	5	15
B2	4	5	3	4	16
B3	3	3	5	3	14
B4	2	2	4	2	10
B5	1	1	2	1	5

table T_2 with variable X_4 reversed

- $Gini(Z(T_2)) = 0.2321$

Table T_2

	X1	rev(X4)	X2	X3	Z
B1	5	5	4	1	15
B2	4	4	5	3	16
B3	3	3	3	5	14
B4	2	2	2	4	10
B5	1	1	1	2	5

table T_2 with variable X_4 reversed, ordered to maximize ρ^*_{max}

- $Gini(Z(T_2)) = 0.2321$
- $\rho^*_{max}(T_2) = 0.1800$

Bank Example – 2nd reversing

Table T_3

	X1	rev(X4)	X2	rev(X3)	Z
B2	4	4	5	3	16
B1	5	5	4	5	19
B3	3	3	3	1	10
B4	2	2	2	2	8
B5	1	1	1	4	7

table T_3 with variable X_4 and X_3 reversed

- $Gini(Z(T_3)) = 0.2133$

Table T_3

	rev(X3)	X1	rev(X4)	X2	Z
B5	4	1	1	1	7
B1	5	5	5	4	19
B4	2	2	2	2	8
B2	3	4	4	5	16
B3	1	3	3	3	10

table T_3 with variable X_4 and X_3 reversed, ordered to maximize ρ^*_{max}

- $Gini(Z(T_3)) = 0.2133$
- $\rho^*_{max}(T_3) = 0.2088$

Bank Example - conclusions

- The complete lack of negative coefficients is the “perfect” state, not always attainable, however we can name it

  the highest possible number of nonnegative correlation coefficients achieved after reversals

- the number of pairs of columns positively correlated increases with each step of reversal
- and it is maximal in the final table

Correlation maps for reversing

- Table T_1 (not reversed) shows **two** variables negatively correlated with the other ones
- Table T_2, with X_4 reversed, has only **one** variable negatively correlated
- table T_3 displays **positively** correlated variables

Table T_1

	X1	X2	X3	X4
X1	1	0.9	-0.3	-1
X2	0.9	1	-0.1	-0.9
X3	-0.3	-0.1	1	0.3
X4	-1	-0.9	0.3	1

Table T_2

X1	rev(X4)	X2	X3
1	1	0.9	-0.3
1	1	0.9	-0.3
0.9	0.9	1	-0.1
-0.3	-0.3	-0.1	1

Table T_3

rev(x3)	X1	rev(X4)	X2
1	0.3	0.3	0.1
0.3	1	1	0.9
0.3	1	1	0.9
0.1	0.9	0.9	1

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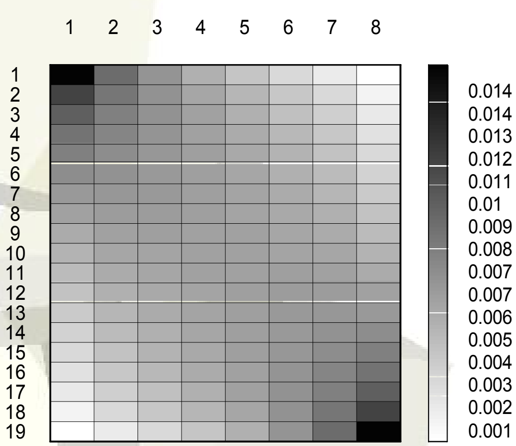
Reversal in a regular model

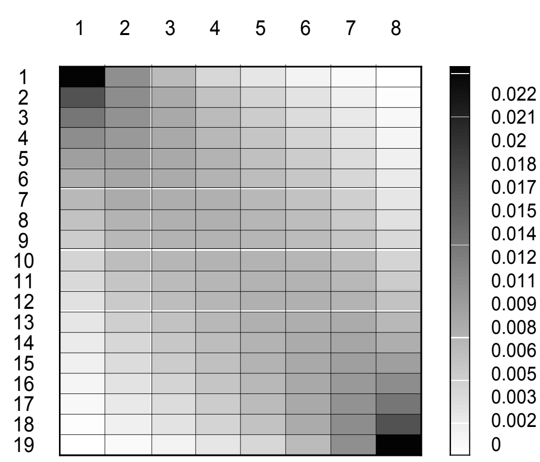
- table 19 rows x 8 columns
- by discretization and aggregation of the distribution of ($\Phi(X)$, $\Phi(Y)$), where:

Φ = cdf of normal distribution N(0,1)
(X, Y) = standard binormal pair: zero means, unit variances, correlation coefficient = 0.3 or 0.5 or 0.7

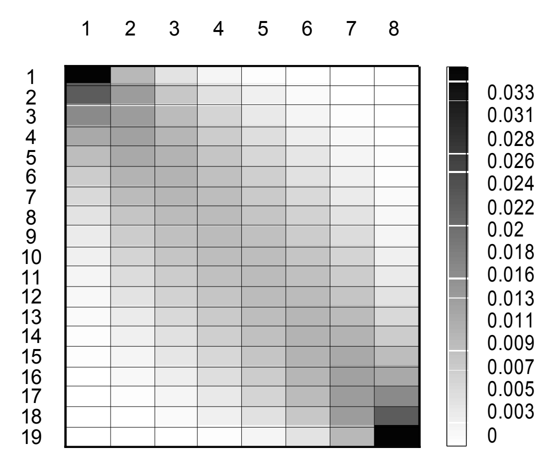
$\rho = 0.3$



$\rho = 0.5$



$\rho = 0.7$



Reversal in a regular model

Pearson correlation maps for $T_j(0.3)$, $j = 0,\ldots,5$

$j = 0$

	X1	X2	X3	X4	X5	X6	X7	X8
X1	1	0.96	0.85	0.42	-0.78	-0.99	-0.97	-0.89
X2	0.96	1	0.96	0.65	-0.59	-0.94	-0.99	-0.97
X3	0.85	0.96	1	0.82	-0.36	-0.82	-0.94	-0.99
X4	0.42	0.65	0.82	1	0.21	-0.36	-0.59	-0.78
X5	-0.78	-0.59	-0.36	0.21	1	0.82	0.65	0.42
X6	-0.99	-0.94	-0.82	-0.36	0.82	1	0.96	0.85
X7	-0.97	-0.99	-0.94	-0.59	0.65	0.96	1	0.96
X8	-0.89	-0.97	-0.99	-0.78	0.42	0.85	0.96	1

$j = 1$

	X1	rev(X8)	X2	X3	X4	X5	X6	X7
X1	1	0.99	0.96	0.85	0.42	-0.78	-0.99	-0.97
rev(X8)	0.99	1	0.96	0.85	0.42	-0.78	-0.99	-0.97
X2	0.96	0.96	1	0.96	0.65	-0.59	-0.94	-0.99
X3	0.85	0.85	0.96	1	0.82	-0.36	-0.82	-0.94
X4	0.42	0.42	0.65	0.82	1	0.21	-0.36	-0.59
X5	-0.78	-0.78	-0.59	-0.36	0.21	1	0.82	0.65
X6	-0.99	-0.99	-0.94	-0.82	-0.36	0.82	1	0.96
X7	-0.97	-0.97	-0.99	-0.94	-0.59	0.65	0.96	1

$j = 2$

	X1	rev(X8)	X2	rev(X7)	X3	X4	X5	X6
X1	1	0.99	0.96	0.96	0.85	0.42	-0.78	-0.99
rev(X8)	0.99	1	0.96	0.96	0.85	0.42	-0.78	-0.99
X2	0.96	0.96	1	0.99	0.96	0.65	-0.59	-0.94
rev(X7)	0.96	0.96	0.99	1	0.96	0.65	-0.59	-0.94
X3	0.85	0.85	0.96	0.96	1	0.82	-0.36	-0.82
X4	0.42	0.42	0.65	0.65	0.82	1	0.21	-0.36
X5	-0.78	-0.78	-0.59	-0.59	-0.36	0.21	1	0.82
X6	-0.99	-0.99	-0.94	-0.94	-0.82	-0.36	0.82	1

$j = 3$

	X1	rev(X8)	X2	rev(X7)	X3	rev(X6)	X4	X5
X1	1	0.99	0.96	0.96	0.85	0.85	0.42	-0.78
rev(X8)	0.99	1	0.96	0.96	0.86	0.85	0.42	-0.78
X2	0.96	0.96	1	0.99	0.96	0.96	0.65	-0.59
rev(X7)	0.96	0.96	0.99	1	0.96	0.96	0.65	-0.59
X3	0.85	0.86	0.96	0.96	1	0.99	0.82	-0.36
rev(X6)	0.85	0.85	0.96	0.96	0.99	1	0.82	-0.36
X4	0.42	0.42	0.65	0.65	0.82	0.82	1	0.21
X5	-0.78	-0.78	-0.59	-0.59	-0.36	-0.36	0.21	1

$j = 4$

	X1	rev(X8)	X2	rev(X7)	X3	rev(X6)	rev(X5)	X4
X1	1	0.99	0.96	0.96	0.85	0.85	0.42	0.42
rev(X8)	0.99	1	0.96	0.96	0.86	0.85	0.42	0.42
X2	0.96	0.96	1	0.99	0.96	0.96	0.65	0.65
rev(X7)	0.96	0.96	0.99	1	0.96	0.96	0.65	0.65
X3	0.85	0.86	0.96	0.96	1	0.99	0.82	0.82
rev(X6)	0.85	0.85	0.96	0.96	0.99	1	0.82	0.82
rev(x5)	0.42	0.42	0.65	0.65	0.82	0.82	1	0.99
X4	0.42	0.42	0.65	0.65	0.82	0.82	0.99	1

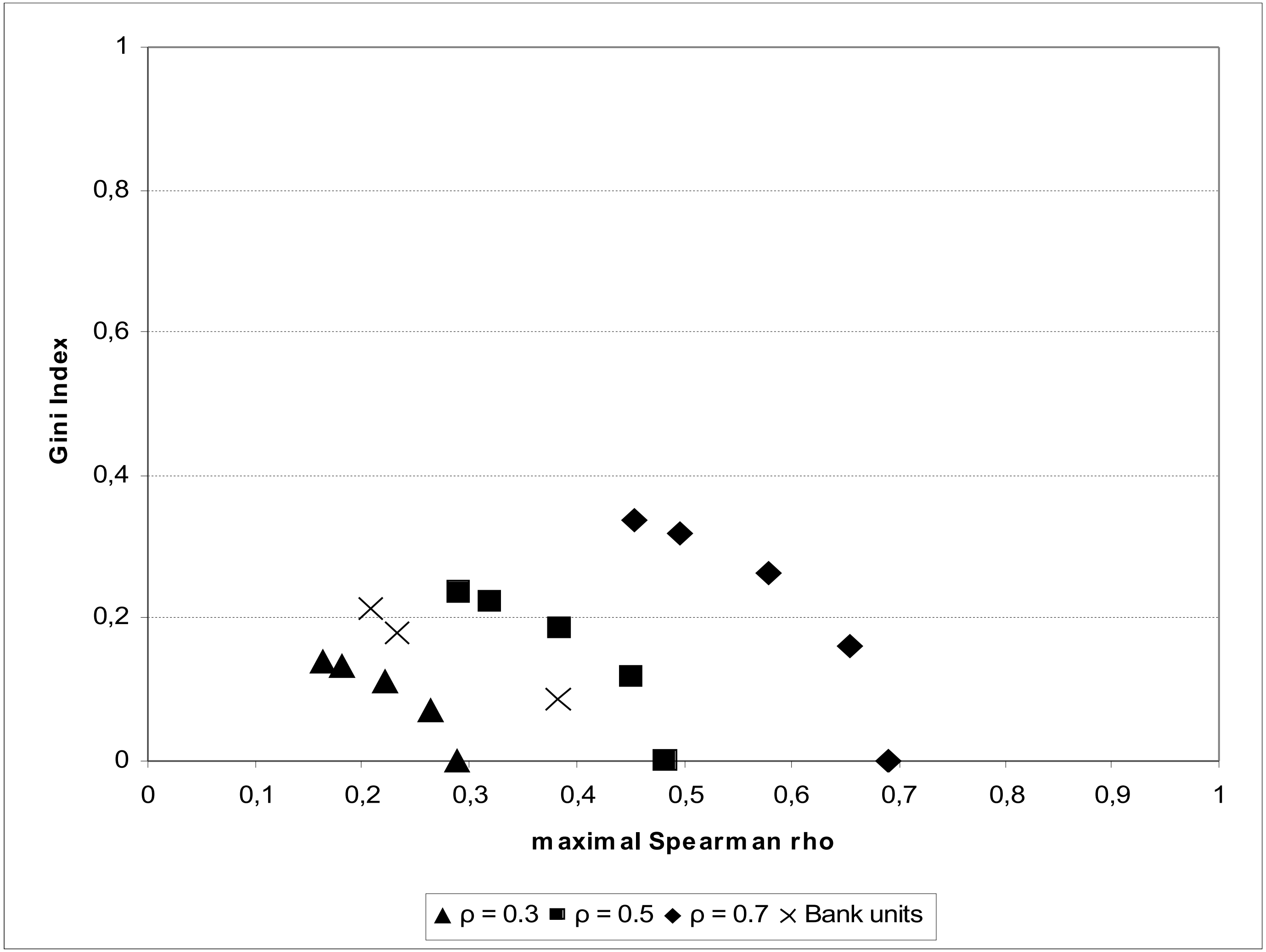
$j = 5$

	X1	rev(X8)	X2	rev(X7)	X3	rev(X6)	rev(X5)	rev(X4)
X1	1	0.99	0.96	0.96	0.85	0.85	0.42	-0.78
rev(X8)	0.99	1	0.96	0.96	0.86	0.85	0.42	-0.78
X2	0.96	0.96	1	0.99	0.96	0.96	0.65	-0.59
rev(X7)	0.96	0.96	0.99	1	0.96	0.96	0.65	-0.59
X3	0.85	0.86	0.96	0.96	1	0.99	0.82	-0.36
rev(X6)	0.85	0.85	0.96	0.96	0.99	1	0.82	-0.36
rev(X5)	0.42	0.42	0.65	0.65	0.82	0.82	1	0.21
rev(X4)	-0.78	-0.78	-0.59	-0.59	-0.36	-0.36	0.21	1

***Gini Index* against ρ^*_{max}**

Step by step reversal of the rightmost columns resulting in

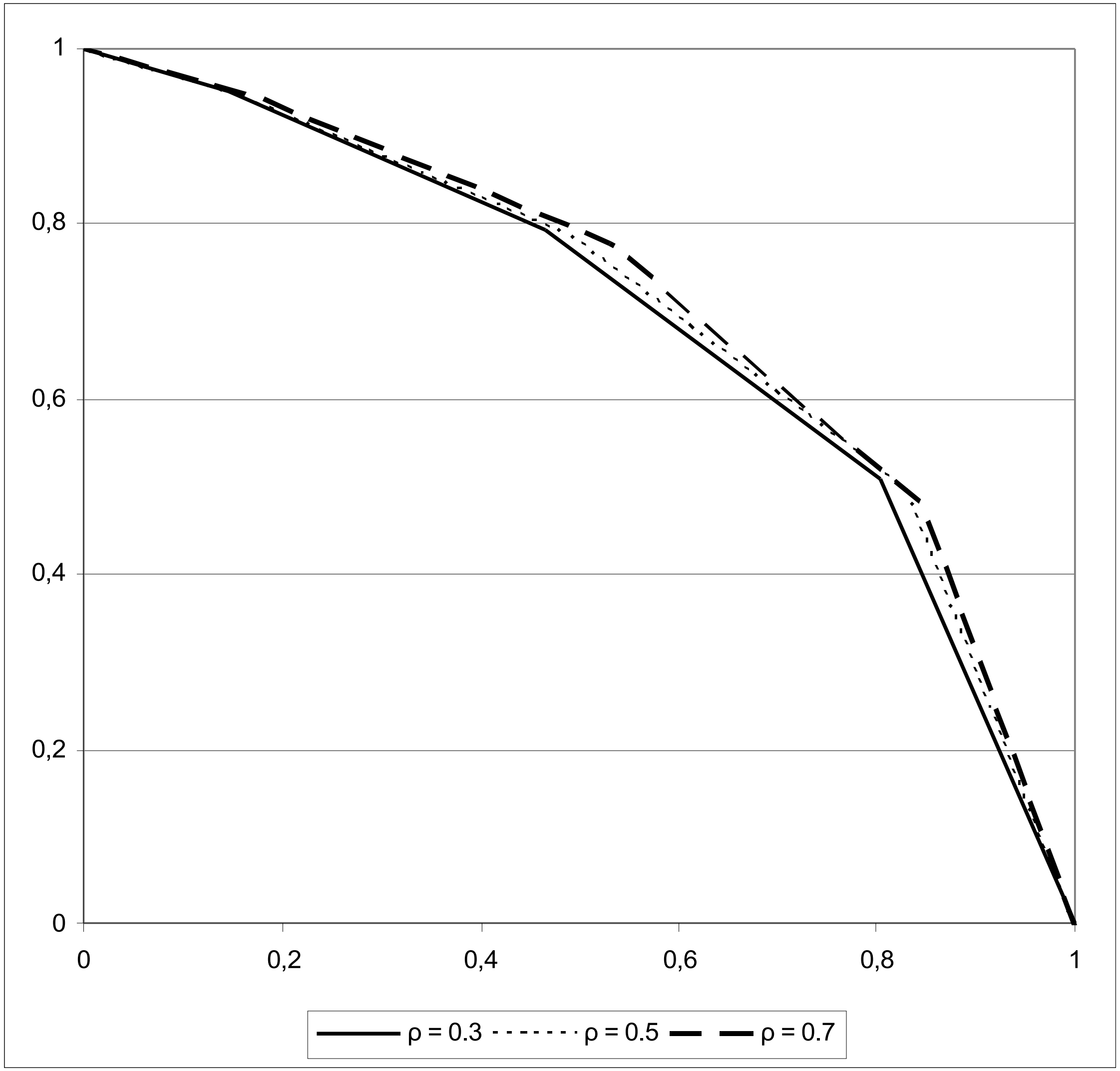
- increasing of *Gini index* (similarity) and
- decreasing of $\rho^*_{max}(T_j)$



The reversing stopped when 4 columns were reversed.

Gini Index against ρ^*_{max}

- The reversal from previous plot, transformed into the unit square
- the points are connected by segments forming linear normalized graphs
- to better show, how they overlap



Regular model - conclusions

- Each quintiple of points seems to lie on the plot of regularly decreasing function
- Therefore: for a fixed ρ dissimilarity measure $\rho^*_{max}(T_j(\rho))$ is **opposite** to the similarity measure *Gini(Z(ρ))*
- „Bank graph” shape outlies from these three graphs, but the departure is not large
- Future plans: Bank Example will be compared with **ranked** data from the regular model – ranks will behave differently than the original ratio data

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Sporting Event Example

- The ranks given by 4 judges (J_1, ... , J_4)
- 19 competitors (c_6, c_7, c_{11}, ... , c_{88}) assessed

The aim:
to diagnose judges assessing participants „almost in reverse to the remaining judges” and to increase variability (to make ranking more meaningful)

Raw data map

	J1	J2	J3	J4	Z
c35	19	18	2	1	40
c76	15	10	1	2	28
c60	16	15	3	3	37
c7	17	17	5	4	43
c28	18	19	8	6	51
c32	12	9	4	5	30
c77	13	14	7	7	41
c9	14	16	10	9	49
c10	8	8	6	8	30
c6	10	11	13	10	44
c87	9	12	16	12	49
c70	11	13	18	15	57
c67	7	6	15	13	41
c82	5	4	9	11	29
c45	6	5	17	16	44
c17	4	7	19	19	49
c11	3	3	11	14	31
c43	2	2	14	17	35
c88	1	1	12	18	32

Sporting Event Example

Table T_1 - initial correlation map

$Gini(Z(T_1)) = 0.1194$

$\rho^*_{max}(T_1) = 0.4540$

	J1	J2	J3	J4
J1	1	0.94	-0.64	-0.87
J2	0.94	1	-0.42	-0.71
J3	-0.64	-0.42	1	0.9
J4	-0.87	-0.71	0.9	1

Table T_2 with J_4 reversed

$Gini(Z(T_2)) = 0.1765$

$\rho^*_{max}(T_2) = 0.3629$

	rev(J4)	J1	J2	J3
rev(J4)	1	0.87	0.71	-0.9
J1	0.87	1	0.94	-0.64
J2	0.71	0.94	1	-0.42
J3	-0.9	-0.64	-0.42	1

Table T_3 with J_4 and J_3 reversed

$Gini(Z(T_3)) = 0.2830$

$\rho^*_{max}(T_3) = 0.2377$

	rev(J3)	rev(J4)	J1	J2
rev(J3)	1	0.9	0.64	0.42
rev(J4)	0.9	1	0.87	0.71
J1	0.64	0.87	1	0.94
J2	0.42	0.71	0.94	1

Table T_4 with reversed: J_4, J_3 and J_2

$Gini(Z(T_4)) = 0.2830$

$\rho^*_{max}(T_4) = 0.3656$

	rev(J2)	rev(J3)	rev(J4)	J1
rev(J2)	1	-0.42	-0.71	-0.94
rev(J3)	-0.42	1	0.9	0.64
rev(J4)	-0.71	0.9	1	0.87
J1	-0.94	0.64	0.87	1

Sporting Event Example

- before reversal
- ordered by GCA

	J1	J2	J3	J4	Z
c35	19	18	2	1	40
c76	15	10	1	2	28
c60	16	15	3	3	37
c7	17	17	5	4	43
c28	18	19	8	6	51
c32	12	9	4	5	30
c77	13	14	7	7	41
c9	14	16	10	9	49
c10	8	8	6	8	30
c6	10	11	13	10	44
c87	9	12	16	12	49
c70	11	13	18	15	57
c67	7	6	15	13	41
c82	5	4	9	11	29
c45	6	5	17	16	44
c17	4	7	19	19	49
c11	3	3	11	14	31
c43	2	2	14	17	35
c88	1	1	12	18	32

- after reversal
- ordered by *Z*

	rev(J3)	rev(J4)	J1	J2	Z
c88	8	2	1	1	12
c43	6	3	2	2	13
c17	1	1	4	7	13
c45	3	4	6	5	18
c11	9	6	3	3	21
c67	5	7	7	6	25
c82	11	9	5	4	29
c70	2	5	11	13	31
c87	4	8	9	12	33
c6	7	10	10	11	38
c10	14	12	8	8	42
c9	10	11	14	16	51
c32	16	15	12	9	52
c77	13	13	13	14	53
c76	19	18	15	10	62
c28	12	14	18	19	63
c60	17	17	16	15	65
c7	15	16	17	17	65
c35	18	19	19	18	74

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Hypermarkets Example

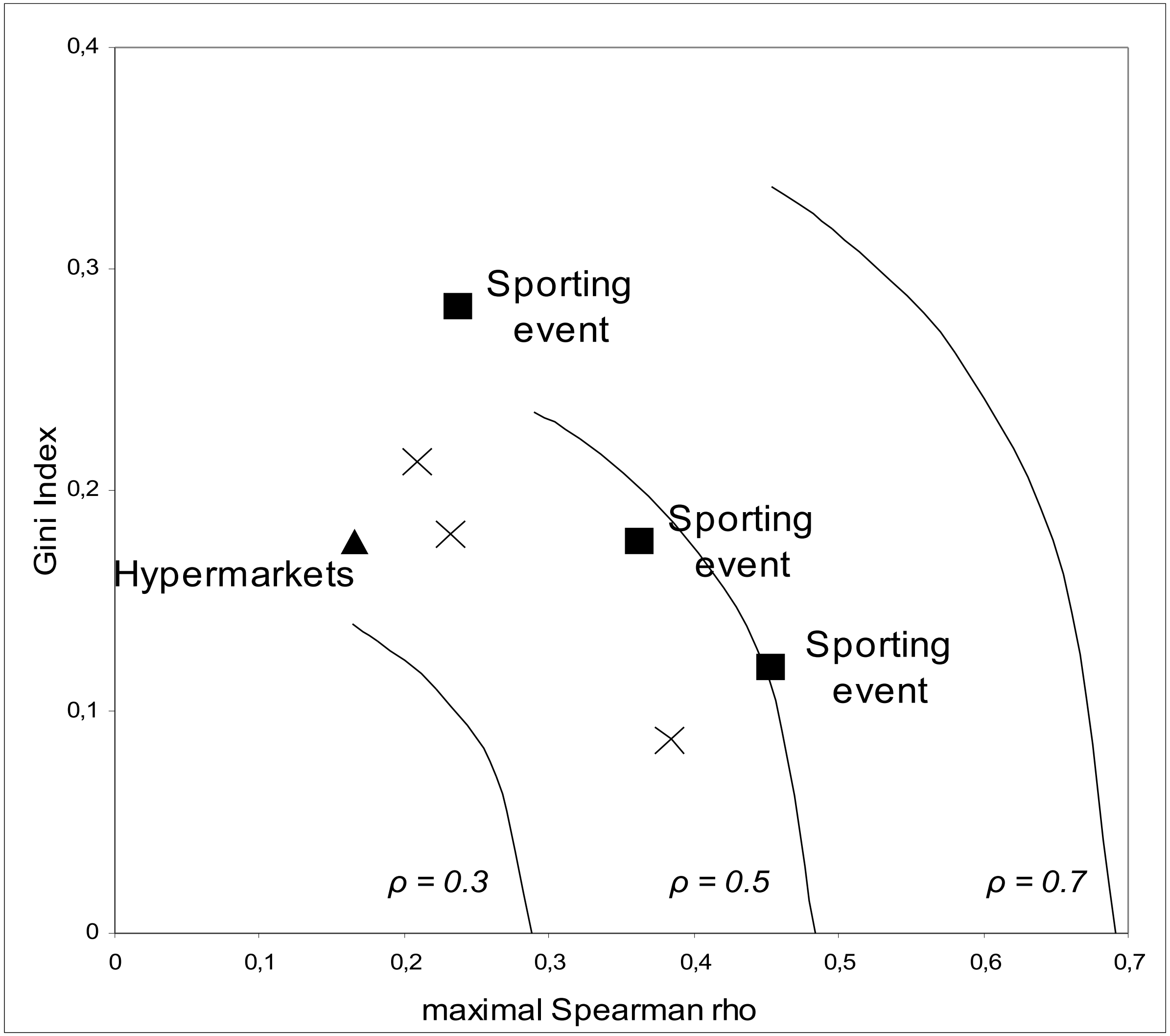
- 19 Finnish hypermarkets (1990)
- 4 performance indicators (*profit before taxes, sales profit, net profit/staff hours, net profit/sales space in m^2*) on rank scale
- all indicators positively correlated
- reversal algorithm did not indicate reversals
- $Gini(Z(T_1)) = 0.1768$
- $\rho^*_{max}(T_1) = 0.1659$

	PROF	SPRO	NPPH	NPPS	Z
Jyvaskyla	1	2	1	1	5
Kouvola	2	4	2	3	11
Kokkola	4	3	3	2	12
Tampere	3	8	6	6	23
Raahe	11	5	4	5	25
Pietarsaari	9	6	10	7	32
Joensuu	7	10	8	11	36
Oulu	5	7	15	9	36
Raisio	14	1	9	13	37
Kotka	8	11	5	14	38
Piispanristi	10	15	7	8	40
Malmi	6	9	17	12	44
Forssa	16	19	13	4	52
Seinajoki	15	12	12	17	56
Turku	13	17	11	16	57
Varkaus	17	13	14	15	59
Pori	12	16	16	18	62
Iisalmi	18	18	18	10	64
Vaasa	19	14	19	19	71

No need for reversal, matrix ordered by *Z* value

Reversing in irregular data sets

- Original graph with irregular models compared with the regular models and the Bank Example
- The irregular models surprisingly well fit the regular, reversed models



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Conclusions

- Reversing provides a new set of traits, with as much of them, as possible strongly positively correlated; thus the sum of new traits have greater variability than before reversing
- Reversal is essential for a proper juxtaposition of total similarity measures with total dissimilarity measures
- Likeness and regularity of the graphs (for various values of ρ) in regular models suggest that we can treat *Gini index* as a very regularly decreasing function of the *maximal Spearman rho* (ρ^*_{max})
- Yet simple reversing is suitable for ranked data or for extremely regular data on a ratio scale

Thank you!

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