

## Link between grade measures of dependence and of separability in pairs of conditional distributions

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### Abstract

Two grade measures of monotone dependence, Spearman's  $\rho^*$  and Kendall's  $\tau$ , can be expressed as weighted averages of monotone Gini separation indices for pairs of conditional distributions of  $Y$  on  $X$ . This fact is used to show an important property of the measures of absolute dependence  $\rho_{max}^*$  and  $\tau_{max}$ , defined, respectively, as the maximal values of  $\rho^*$  and  $\tau$  over the set of pairs of all the possible one-to-one Borel-measurable transformations of  $X$  and of  $Y$ . Namely, if  $(X, Y)$  are totally positive of order two ( $TP_2$ ) then  $\rho^*(X, Y) = \rho_{max}^*(X, Y)$  and  $\tau(X, Y) = \tau_{max}(X, Y)$ . Moreover, another index  $\tau_{abs}(X, Y)$  of absolute dependence is introduced as weighted average of Gini (absolute) separation indices for the pairs of conditional distributions of  $Y$  on  $X$ . Indices  $\tau_{abs}$  and  $\tau_{max}$  are used to measure the irregularity of dependence. All facts proved in this paper hold for the general case of the mixed discrete-continuous variables.

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